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**ABSTRACT**

The application of piezoelectric actuators for static shape control composite plate is investigated in this thesis. Electro-mechanically coupled mathematical model is used for the analysis. The major section of this thesis is the static shape control. Shape control is defined here as the determination of shape control parameters, including actuation voltage and actuator orientation configuration, such that the structure that is activated using these parameters will conform as close as possible to the desired shape. A finite element model for shape control analysis of piezoelectric laminated composite plate using Ansys is presented in this thesis. Piezoelectric actuators and sensors are modeled as additional layers either to be surface bonded or embedded in the laminated composite plate. A finite element software Ansys is used to model and was successfully validated with experimental and numerical results that are readily available in the literatures. The present analysis shows that with the application of appropriate voltage to piezoelectric actuator, desired shape of the composite plate can be obtained.

**KEYWORDS:** composite plate, nonlinear, piezoelectric material.

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**INTRODUCTION**

The needs for structure with the self-monitoring and self-controlling capabilities especially in aerospace application have caused remarkable growth in the research and development of smart structure. A smart structure can be defined as a structure made up of purely elastic materials, called the substrate, integrated with surface mounted or embedded sensors and actuators that have capability to sense and take corrective action Wang et al(1997). The direct piezoelectric effect is the ability to generate electrical charge in proportion to externally applied mechanical force, and the converse piezoelectric effect is exactly the inverse of the direct effect. Laminated composite plate is chosen as substrate for its high strength to weight and stiffness to weight ratios. These characteristics make the laminated composite plate suitable to be used in many applications especially in aerospace. A composite is a structural material that consists of two or more combined constituents that are combined at a macroscopic level and are not soluble in each other.

One constituent is called the reinforcing phase and the one in which it is embedded is called the matrix. The reinforcing phase material may be in the form of fibers, particles, or flakes. The matrix phase materials are generally continuous. Examples of composite systems include concrete reinforced with steel and epoxy reinforced with graphite fibers and wood, where the lignin matrix is reinforced with cellulose fibers and bones in which the bone-salt plates made of calcium and phosphate ions reinforce soft collagen. etc. [1]

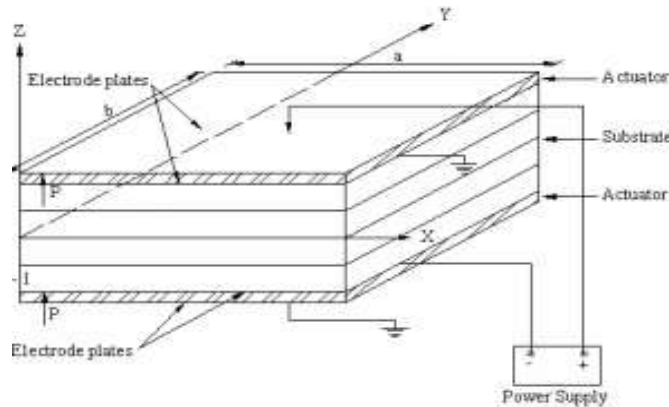
**PIEZOELECTRICAL EFFECT**

A piezoelectric substance is one that produces an electric charge when a mechanical stress is applied (the substance is squeezed or stretched). Conversely, a mechanical deformation (the substance shrinks or expands) is produced when an electric field is applied. This effect is formed in crystals that have no center of symmetry. To explain this, we have to look at the individual molecules that make up the crystal. Each molecule has polarization, one end is more negatively charged and the other end is positively charged, and is called a dipole. This is a result of the atoms that make up the molecule and the way the molecules are shaped. The polar axis is an imaginary line that runs through the center of both charges on the molecule. In a monocrystal the polar axes of all of the dipoles lie in one direction. The crystal is

said to be symmetrical because if you were to cut the crystal at any point, the resultant polar axes of the two pieces would lie in the same direction as the original.

## MATHEMATICAL MODELING

This chapter includes constitutive equation of piezoelectric field and governing Equation of finite element formulation for the intelligent structure.



**Fig 1. Piezoelectric effect.**

### 2.1 PIEZOELECTRICAL FIELD

The linear constitutive equations coupling between elastic field and electric field in a piezoelectric medium can be expressed by the direct and inverse piezoelectric equations, respectively.

These equations for the plate shape sensor and actuator are written as follows,

$$\begin{Bmatrix} D_x \\ D_y \\ D_z \end{Bmatrix} = \begin{pmatrix} e_{11} & e_{12} & e_{16} \\ e_{21} & e_{22} & e_{26} \\ e_{31} & e_{32} & e_{36} \end{pmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \end{Bmatrix} + \begin{pmatrix} \zeta_{11} & \zeta_{12} & \zeta_{13} \\ \zeta_{21} & \zeta_{22} & \zeta_{23} \\ \zeta_{31} & \zeta_{32} & \zeta_{33} \end{pmatrix} \quad (1)$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{Bmatrix} = \begin{pmatrix} D_{p11} & D_{p12} & D_{p13} \\ D_{p21} & D_{p22} & D_{p23} \\ D_{p31} & D_{p32} & D_{p33} \end{pmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \end{Bmatrix} - \begin{pmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{pmatrix} \begin{Bmatrix} E_x \\ E_y \\ E_z \end{Bmatrix} - \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{Bmatrix} \Delta T \quad (2)$$

## RESULTS AND DISCUSSIONS

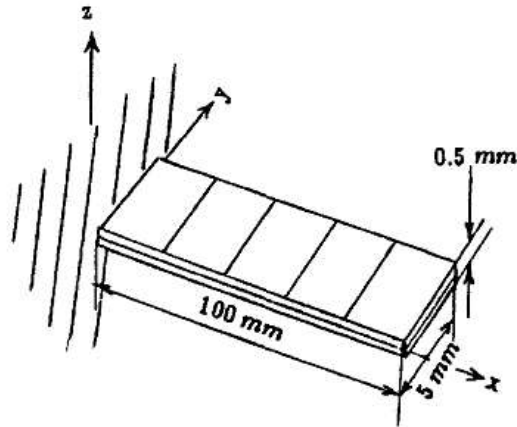
### Validation of the model

The FE model developed in the earlier chapter is first validated and used in the further analysis. In order to verify the accuracy of present model a bimorph piezoelectric beam shown in fig 2 is first studied by Z Wang et al.(1997).

This beam consists of two identical PVDF uniaxial beam with opposite polarities. The cantilever beam is modeled by five identical plate elements. The material properties of PVDF are listed in table1.

The theoretical solution to the deflection of the beam given by Tzou H. S (1989)

$$w(x) = 0.375 \frac{e_{31} V_a}{E} \left( \frac{x^2}{l^2} \right) \quad (3)$$



**Fig 2. Piezoelectric PVDF bimorph beam.**

The deflection of the beam is calculated for various distances between 20mm to 100mm along the length of beam. The results are shown in Tables 2 along with results of Tseng [1990]. The results show the close agreement between theoretical and the present finite element solutions.

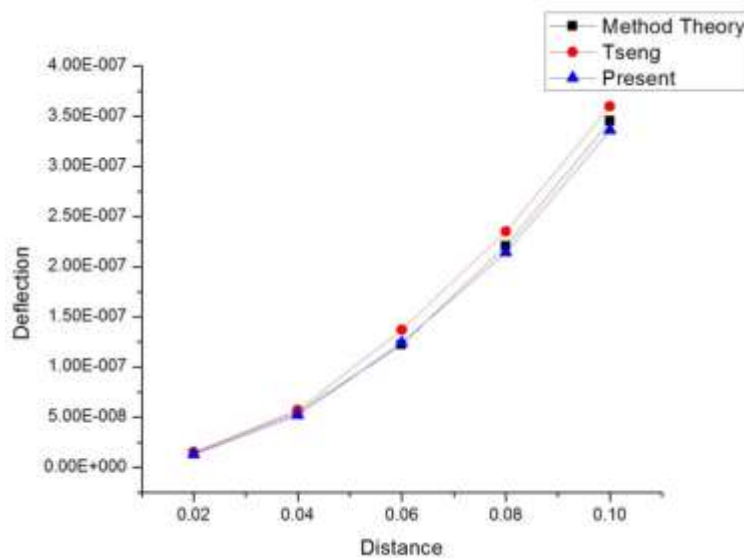
**Table-1: Material properties of the main structure and piezoelectric**

Property	PVDF	Graphite/Epoxy
E <sub>1</sub>	0.2e10 N/m <sup>2</sup>	0.98e11 N/m <sup>2</sup>
E <sub>2</sub>	0.2e10 N/m <sup>2</sup>	0.79e10 N/m <sup>2</sup>
G <sub>12</sub>	0.775 e9 N/m <sup>2</sup>	0.56e10 N/m <sup>2</sup>
ν <sub>12</sub>	0.29	0.29
ν <sub>21</sub>	0.28	0.28
ρ	1800kg/m <sup>3</sup>	1520 kg/m <sup>3</sup>
e <sub>31</sub>	0.046c/m <sup>2</sup>	0
e <sub>32</sub>	0.046c/m <sup>2</sup>	0
e <sub>33</sub>	0	0
ε <sub>11</sub>	0.1062e-9F/m	0
ε <sub>22</sub>	0.1062e-9F/m	0

$\epsilon_{33}$	0.1062e-9F/m	0
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**Table-2: Deflection of PVDF Bimorph Beam (for a unit voltage) (m)**

Distance(m)	RPIM Theory	Tseng(1990)	Present FEM
0.02	0.0140e-6	0.0150e-6	0.0130e-6
0.04	0.0552e-6	0.0569e-6	0.0523e-6
0.06	0.1224e-6	0.1371e-6	0.124e-6
0.08	0.2208e-6	0.2351e-6	0.214e-6
0.1	0.3451e-6	0.3598e-6	0.336e-6



**Fig 3. Deflection of PVDF Bimorph Beam (for a unit voltage) (m)**

### CANTILEVER PLATE WITH UDL

A cantilevered laminated composite plate is studied with both upper and lower layer symmetrically bonded by piezoelectric ceramics. The plate consists of four composite layers and two piezolayers. Plate is made of T300/976 graphite -epoxy composites and piezoceramic is PZT G 1195N.

The material properties are given in Table 3.

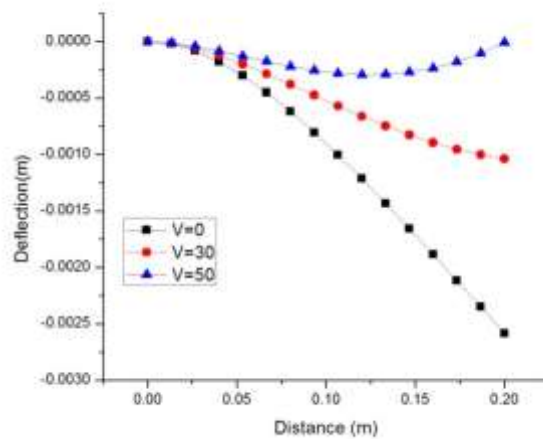
**Table 3: Material Property of PZT G1195N Piezoceramics and T300/976 Graphite-Epoxy Composites**

Property	PZT	T300/976
$E_{11}$	63e9	150e9
$E_{22}$	63e9	9e9
$E_{33}$	63e9	9e9
$\nu_{12}$	0.3	0.3
$\nu_{13}$	0.3	0.3
$\nu_{23}$	0.3	0.3
$G_{12}$	24.2e9	7.10e9
$G_{13}$	24.2e9	2.5e9

$G_{23}$	24.2e9	7.10e9
$\rho$	7600 kg/m <sup>3</sup>	1600 kg/m <sup>3</sup>
$d_{31}$	22.86	-
$d_{32}$	22.86	-
$K_{11}$	15.3e-9 F/m	-
$K_{22}$	15.3e-9	-
$K_{33}$	15e-9	-

Consider composite plate [P/-45/45]<sub>as</sub> is originally flat and is then exposed to uniformly distributed load of 100 N/m<sup>2</sup>. To flatten the plate an active voltage is added incrementally until center line deflection of plate is reduced to desired limits.

For the present analysis, dimensions of the plate are (0.2m x 0.2m). The plate consists of four composite layers and two outer piezo layers. Total thickness of non-piezoelectric composite plate is 0.001m and thickness of piezo layer is 0.0005m.



**Fig 4. The centerline deflection of cantilever laminate [p/-45/45]<sub>as</sub> under uniform loading and different actuator input voltages.**

In static analysis all the piezoceramics on the upper and lower surfaces of plate are used as actuator or sensors. When equal amplitude voltages with opposite sign are applied across the thickness of the two piezo layers they will contract or expand depending on whether applied voltage is negative or positive and consequently strains are induced to generate forces that bend composite plate.

## EFFECT OF INCREASING NUMBER OF LAYERS

### A. Cross Ply Laminates for Cantilever Plate

The anti-symmetric/symmetric cross ply plates subjected to the uniformly distributed loads and different voltages are applied to study the behavior and the effect of layer on nonlinear centerline deflection. The Fig 5-16 presents the plots of centerline deflection versus distance for different number of layer and it can be observed that, as the number of layer increases the centerline deflection decreases.

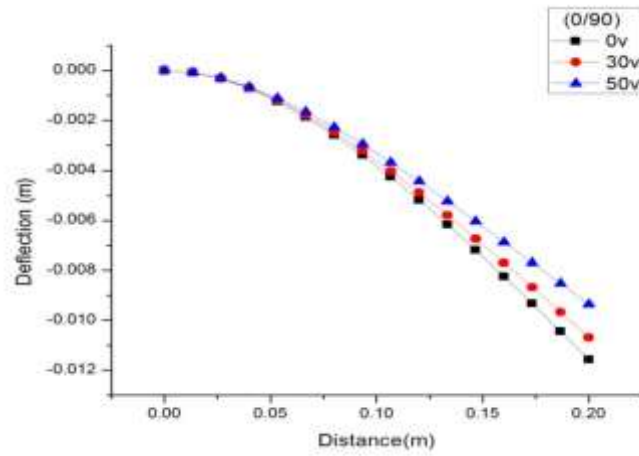


Fig 5. Effects on numbers of layer on Centerline deflection for cross ply (0/90) subjected to the load  $P=100N/m^2$  and different voltages.

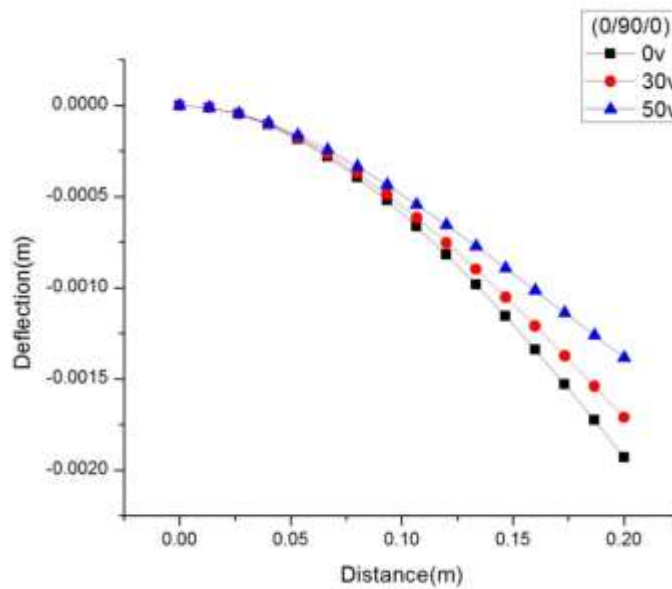


Fig 6: Effects on numbers of layer on Centerline deflection for cross ply (0/90/0) subjected to the load  $P=100N/m^2$  and different voltages.

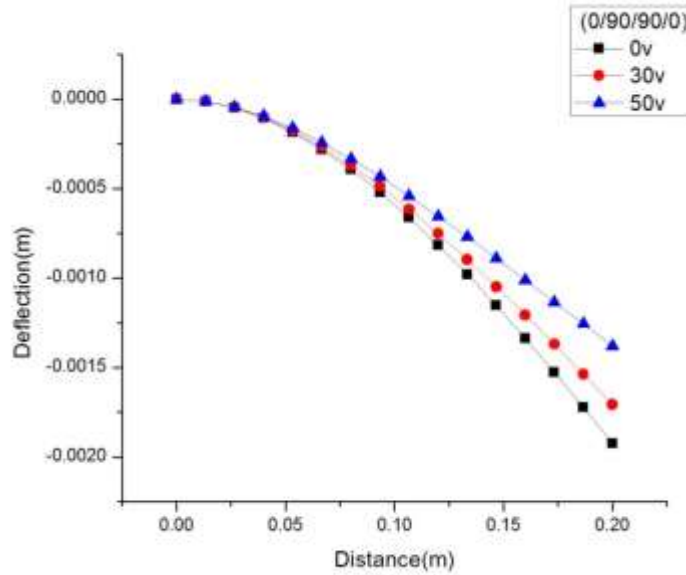


Fig 7: Effects on numbers of layer on Centerline deflection for cross ply (0/90/90/0) subjected to the load  $P=100N/m^2$  and different voltages.

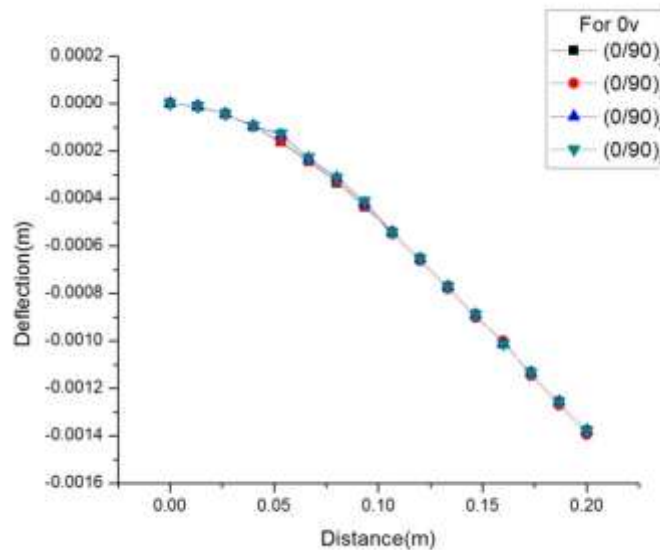
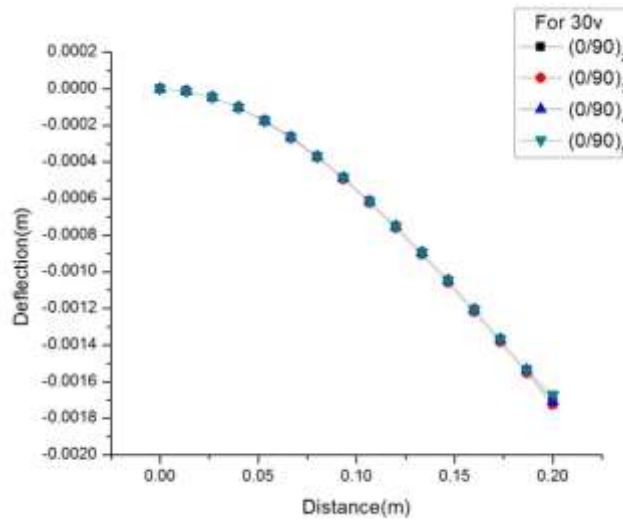
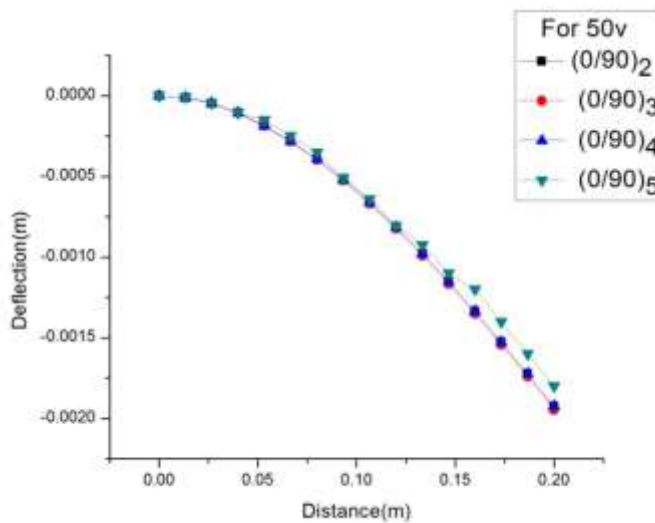


Fig 8: Effects on numbers of layer on centerline deflection for cross ply subjected to the load  $P=100N/m^2$  and zero voltages.



**Fig 9: Effects on numbers of layer on centerline deflection for cross ply subjected to the load  $P=100N/m^2$  and 30 voltage.**



**Fig 10: Effects on numbers of layer on centerline deflection for cross ply subjected to the load  $P=100N/m^2$  and zero voltages.**

**B. Cross ply laminates for clamped Plate**



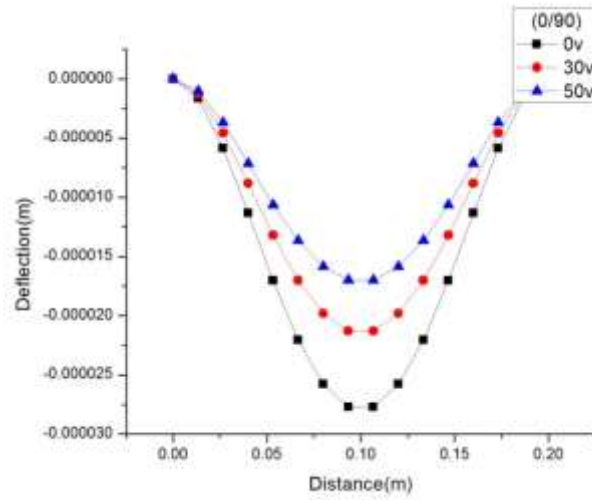


Fig 11. Effects on numbers of layer on centerline deflection for cross ply subjected to the load  $P=100N/m^2$  and different voltages.

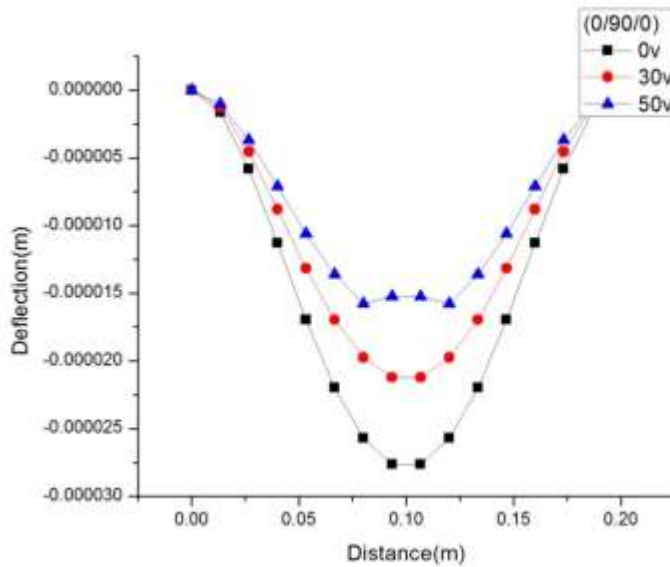


Fig 12. Effects on numbers of layer on centerline deflection for cross ply subjected to the load  $P=100N/m^2$  and different voltages.

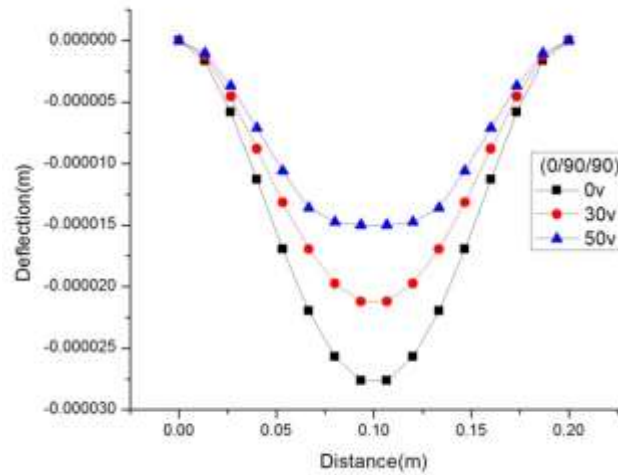


Fig 13. Effects on numbers of layer on centerline deflection for cross ply subjected to the load  $P=100\text{N/m}^2$  and different voltages.

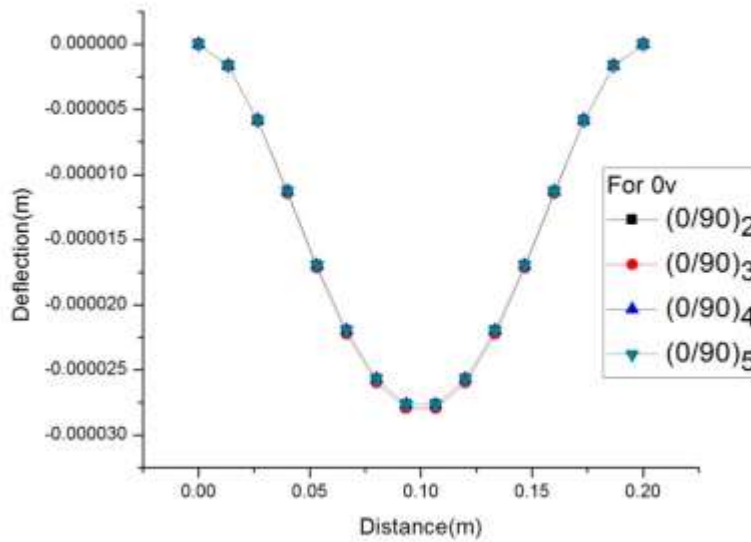


Fig 14. Effects on numbers of layer on centerline deflection for cross ply subjected to the load  $P=100\text{N/m}^2$  and zero voltages.

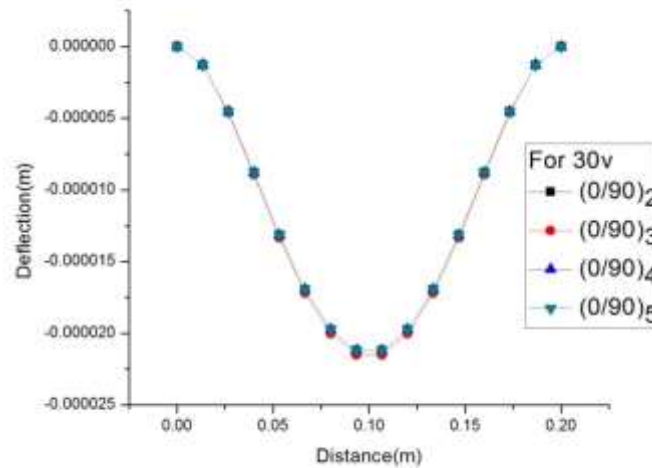


Fig 15. Effects on numbers of layer on centerline deflection for cross ply subjected to the load  $P=100\text{N/m}^2$  and 30voltage.

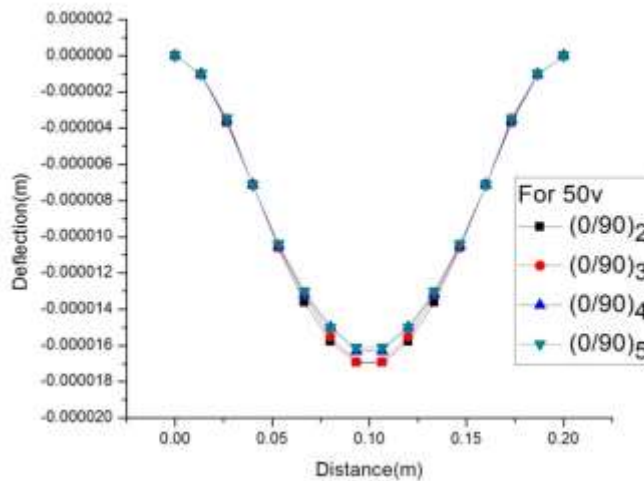


Fig 16. Effects on numbers of layer on centerline deflection for cross ply subjected to the load  $P=100\text{N/m}^2$  and 50voltage.

## CONCLUSIONS

A finite element formulation for the plate with distributed piezoelectric sensors/actuators is presented. A piezoelectric plate element is developed. Based on the plate element, a general method was developed for the static shape control of the intelligent structure. The input voltage and feedback gain making the shape of the intelligent structure reach the desired shape can be obtained by the present method. The behavior of a piezoelectric laminated simply supported plate is studied. As a summary, the behaviors of the beam subjected to electric and mechanical loading are listed below:

1. The deflections of the plate decreases as applied voltage increases.
2. It would be concluded that the deflections due to mechanical loading can be effectively controlled by applying appropriate voltages to the piezolayers and can be said that piezoelectric layers are useful in controlling deflections of plate under electro mechanical loadings.
3. As the number of layers increases, centerline deflection goes on decreasing.

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